Noise in transmission systems

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- Noise in transmission systems
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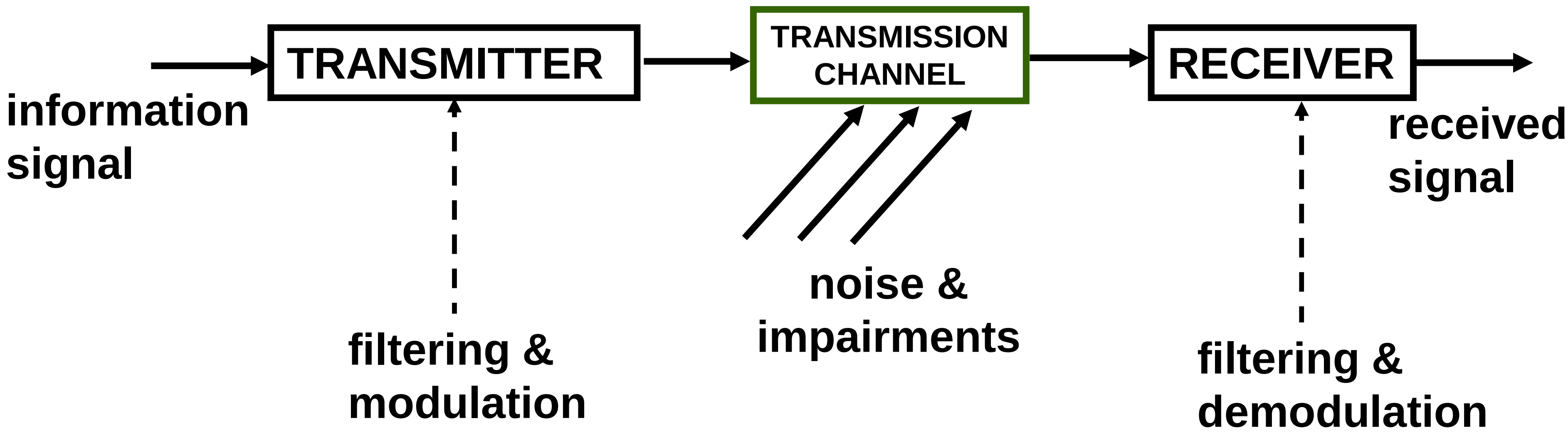
Transmission System



__Transmitter__ is an electronic device which aims at sending information signals to a receiver.

__Receiver__ is an electronic device that receives transmitted signals and converts the information carried by them to a usable form.

Most important processing (from signal theory point of view) is __modulation__, __demodulation__ (detection) and __filtering__.

Noise vs. Impairments

<u>Noise is a random fluctuation</u> in an electrical signal and it can be produced by several different effects.
Noise is a summation of unwanted or disturbing energy from natural sources.

<u>Impact of noise can be minimized to some extent.</u>

<u>Impairments</u> are distinguished from noise as:
- electromagnetic interference from other transmitters,
- distortion being an unwanted systematic alteration of the signal waveform by the communication equipment (nonlinearity, fading, dispersion).

<u>Impairments can be (almost) completely eliminated.</u>

Noise sources in a transmission system

<u>Two broad classes</u> of noise sources:
- external to the system,
- internal to the system.

<u>External noise sources:</u>
- atmospheric noise,
- solar noise,
- cosmic noise.

<u>Internal noise sources:</u>
- <u>thermal noise</u> (sz. termiczny),
- shot noise (sz. śrutowy),
- generation-recombination noise,
- flicker noise (sz. migotania),
- quantum noise (sz. kwantowy).

<u>Thermal noise</u> (chaotic fluctuation of charge carriers) is present in all electrical circuits and can drown out weak signals thus being the limiting factor of telecommunication systems performance.

Thermal noise (AWGN)

1. Thermal noise (Nyquist noise) is the electronic noise generated by the thermal excitation of the charge carriers (usually the electrons) inside an electrical conductor which happens regardless of any applied voltage.

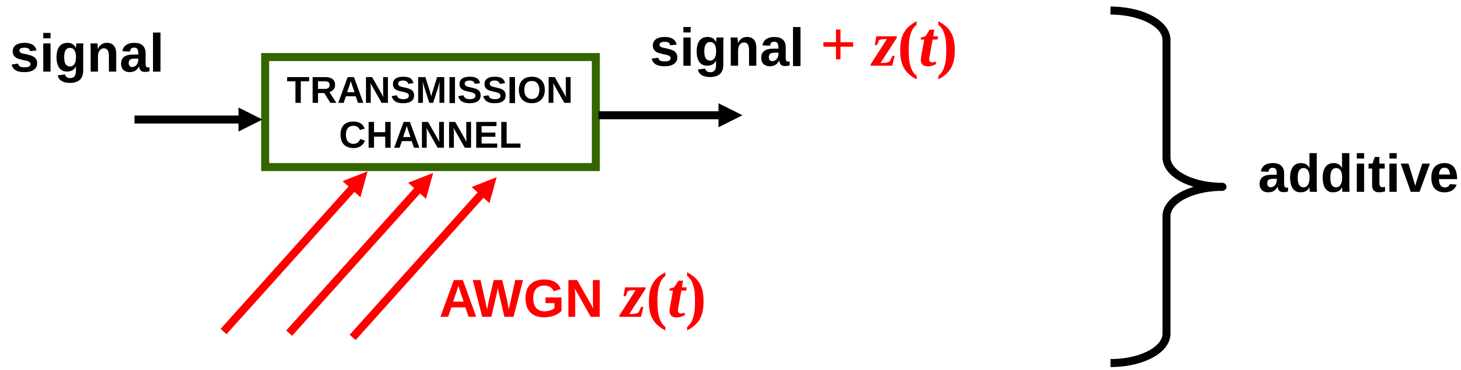
2. Thermal noise amplitudes are assumed to be governed by a Gaussian (normal) pdf. The assumption is justified by the central-limit theorem as the noise is a result of random motion of a large number of charge carriers.

3. Thermal noise power spectrum is flat (white) up to frequencies of 1000 GHz = 1 THz (at *T* = 290 degK= 17 degC).

4. Thermal noise is assumed to be added to signals transmitted over a channel.

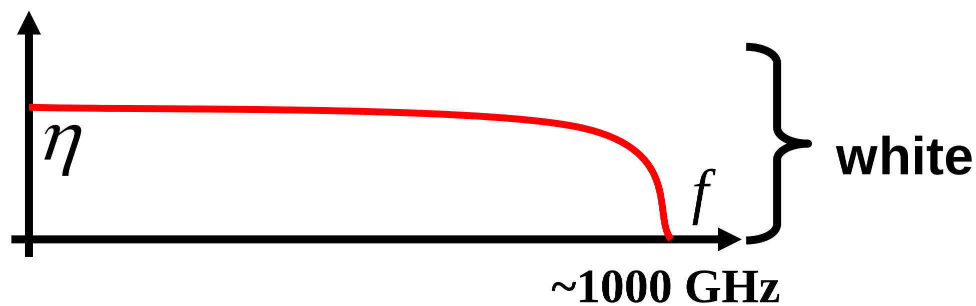
4 & 3 & 2 & 1 = Additive White Gaussian Noise (AWGN)
White = Wideband

Additive White Gaussian Noise



$$S_z(f)[\mathrm{W/Hz}] \approx 4kT\,[\mathrm{W/Hz}]$$

$$S_z(\omega)[\mathrm{W/[rad/s]}] = \eta(= \pi kT)[\mathrm{W/[rad/s]}]$$

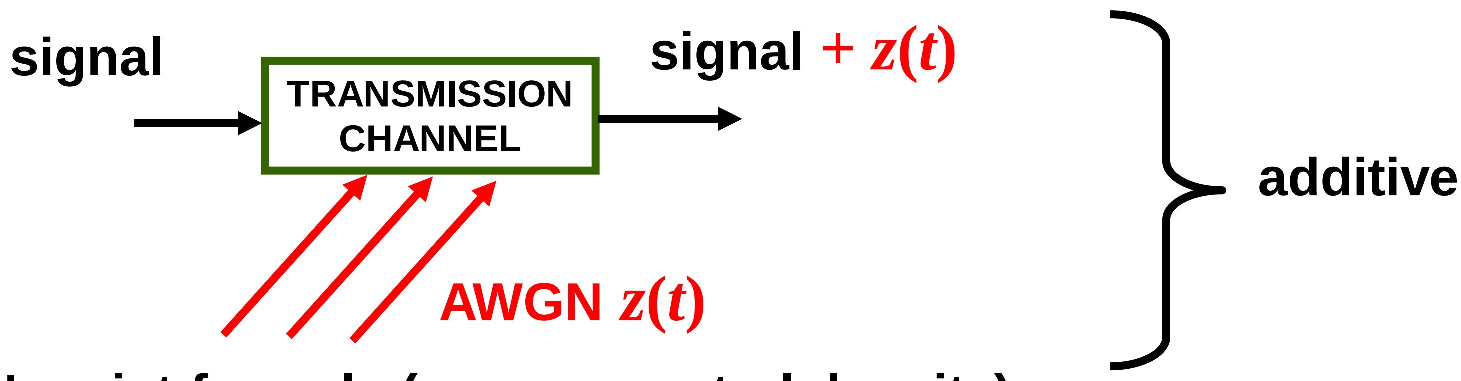


$$k = 1.38\times 10^{-23}\ \mathrm{J/degK}\ \text{(Boltzman's constant)}$$

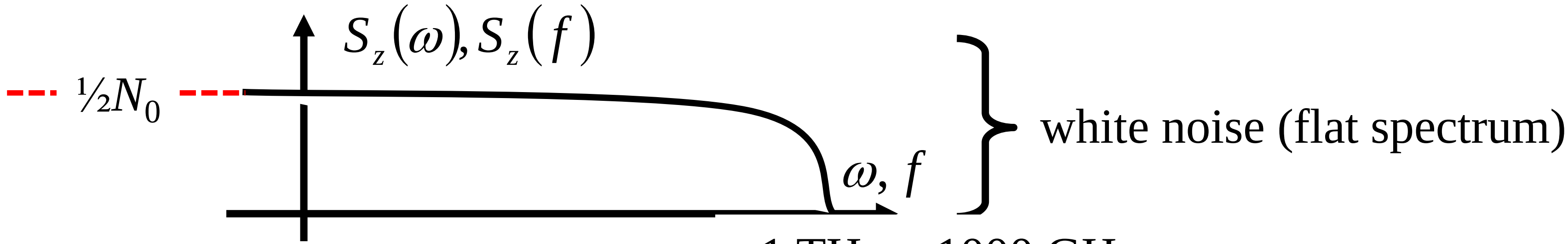
$$T[\mathrm{degK}] - \text{temperature}$$

$$R = 1\,\Omega$$

Additive White Gaussian Noise



Johnson-Nyquist formula (power spectral density)



~1 THz = 1000 GHz

$$S_z(f)[\mathrm{Wat/Hz}] = 4kT = \frac{1}{2}N_0$$

$$S_z(\omega)\left[\frac{\mathrm{Wat}}{\mathrm{rad/s}}\right] = \frac{4kT}{2\pi} = \frac{2kT}{\pi} = \frac{1}{2}N_0$$

$k = 1.38 \times 10^{-23}$ J/degK (Boltzman constant)

$T[\mathrm{degK}]$ – temperature

$R = 1\Omega$

AWGN power increases with a temperature as fluctuations of electrons grow as well.

Logaritmic measure of AWGN power

$$\text{deciBel} - \text{Wat}:$$

$$N\,[\text{dB}] = N\,[\text{dBW}] = 10\,\lg\left(N\,[\text{W}]/1\,\text{W}\right)$$

$$\text{deciBel} - \text{miliWat}:$$

$$N\,[\text{dBm}] = 10\,\lg\left(N\,[\text{W}]/1\,\text{mW}\right)$$

$$\text{dBW vs dBm}$$

$$N\,[\text{dBm}] = N\,[\text{dB}] + 30$$

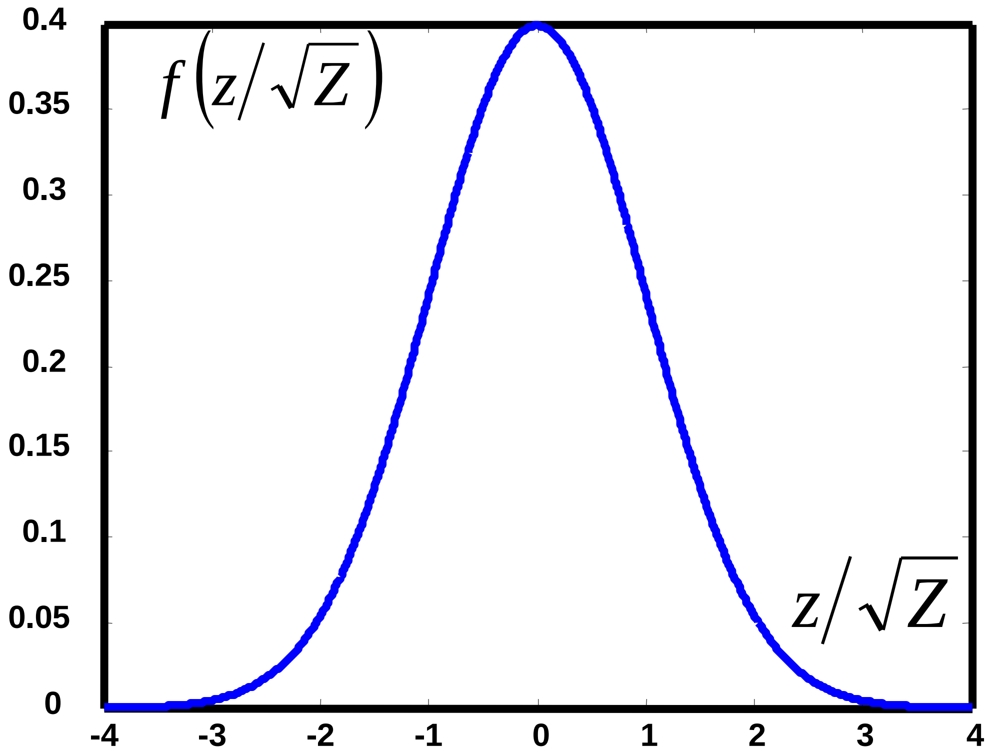
Power in dBm – some examples

80 dBm	100 kW	Typical transmission power of FM radio station
50 dBm	100 W	Typical thermal radiation emitted by a human body
0 dBm	1 W	Typical RF leakage from a microwave oven
− 80 dBm	10pW	Typical range of wireless received signal power
−106 dBm		Analog television channel
−127.5 dBm		Typical received signal power from a GPS satellite
−134 dBm		AWGN in a 10 kHz bandwidth
−174 dBm		AWGN in a1 Hz bandwidth

Additive White Gaussian Noise

$$f(z)=\frac{1}{\sqrt{2\pi Z}}e^{-\frac{z^2}{2Z}}=\frac{1}{\sqrt{2\pi Z}}\exp\left(-\frac{z^2}{2Z}\right)=\frac{1}{\sqrt{2\pi Z}}\exp\left(-\frac{\left(z/\sqrt{Z}\right)^2}{2}\right)$$

$$\overline{z}=0,\ \overline{z^2}=Z=\frac{1}{\pi}\int_W S(\omega)d\omega=\eta W/\pi\,[\mathrm{W}]$$



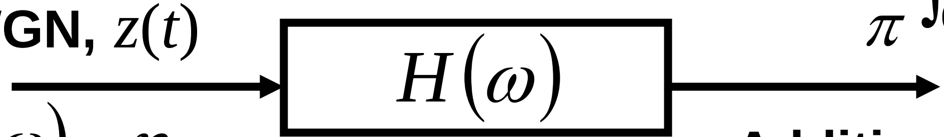
Normalized Gaussian distribution

Filtering AWGN

$$S_n(\omega) = \eta |H(\omega)|^2$$

$$N = \frac{1}{\pi} \int_0^{\infty} \eta |H(\omega)|^2 \, d\omega$$

AWGN, $z(t)$



$S_z(\omega) = \eta$

Additive Gaussian noise, $n(t)$

Any kind of filtering does preserve the Additive Gaussian form of p.d.f.s.

$$f(n) = \frac{1}{\sqrt{2\pi N}} e^{-\frac{n^2}{2N}} = \frac{1}{\sqrt{2\pi N}} \exp\left(-\frac{n^2}{2N}\right)$$

$$\overline{n} = 0, \overline{n^2} = N$$

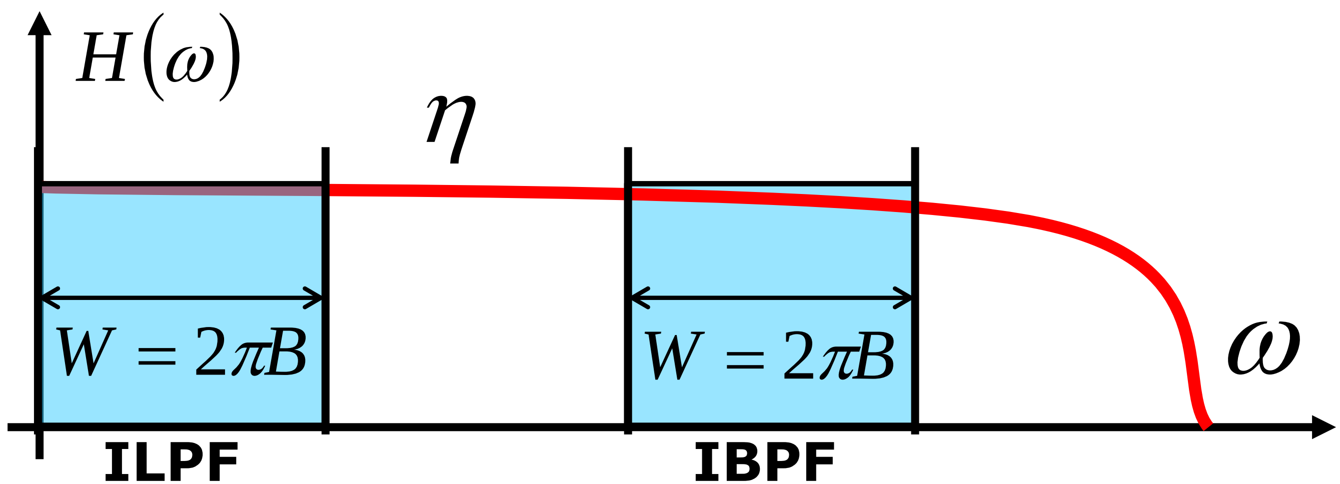
Filtering AWGN

Ideal low (ILPF)- or bandpass (IBPF) filter $H(\omega)$

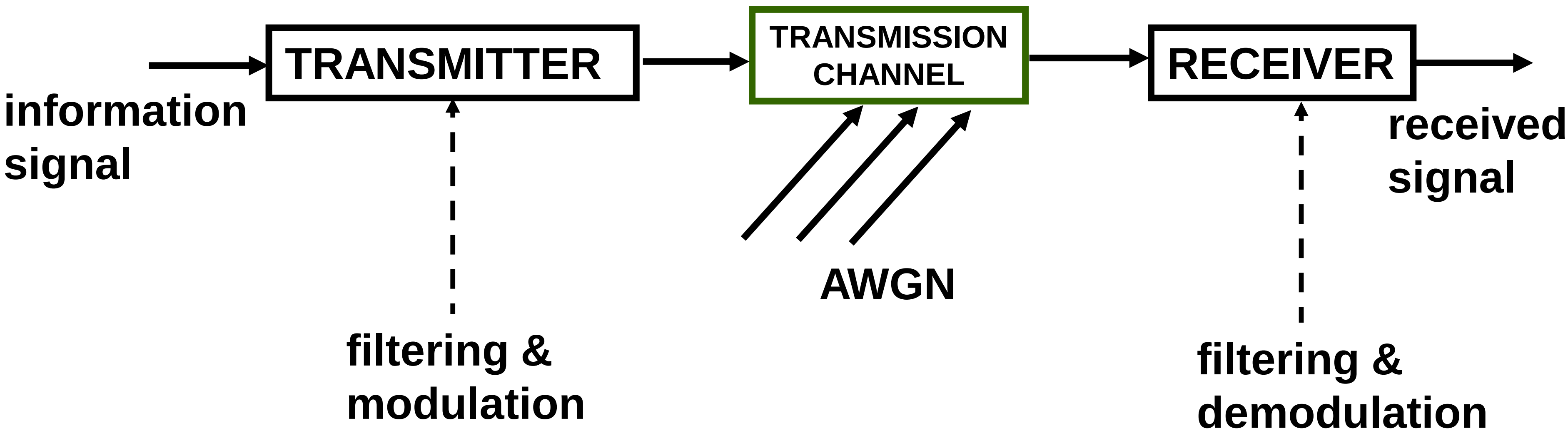
AWGN, $z(t)$ **(wideband)** → ILPF/IBPF $\boxed{H(\omega)}$ → **ANGN,** $n(t)$ **(narrowband)**

$$S_z(\omega) = \eta$$

$$N = \frac{1}{\pi}\int_W \eta \left|H(\omega)\right|^2 d\omega = \frac{\eta W}{\pi} = 2\eta B$$

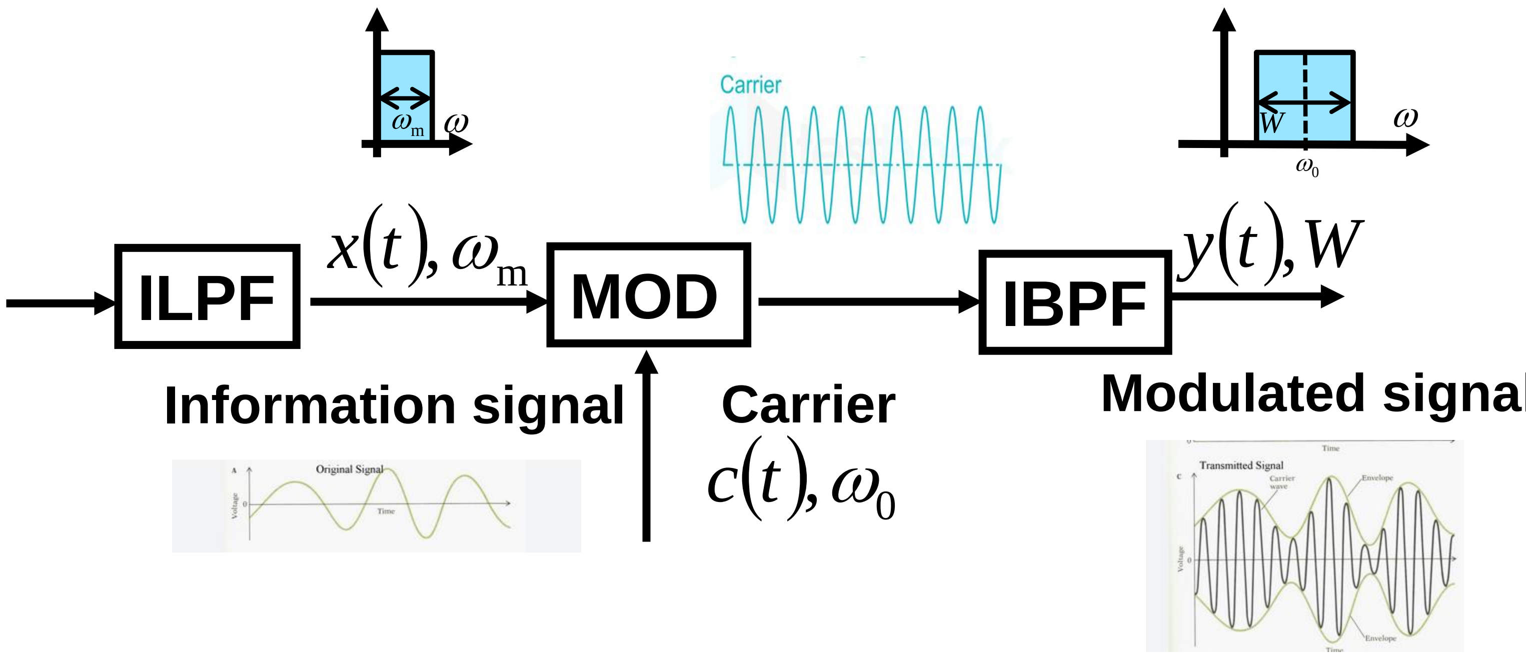


Noise performance of a transmission system



<u>AWGN - thermal fluctuations of the charge carriers</u> (usually the electrons) inside any electrical circuit which happens regardless of any applied voltage. AWGN is modeled as a Gaussian random proces with a flat (white) power distribution.

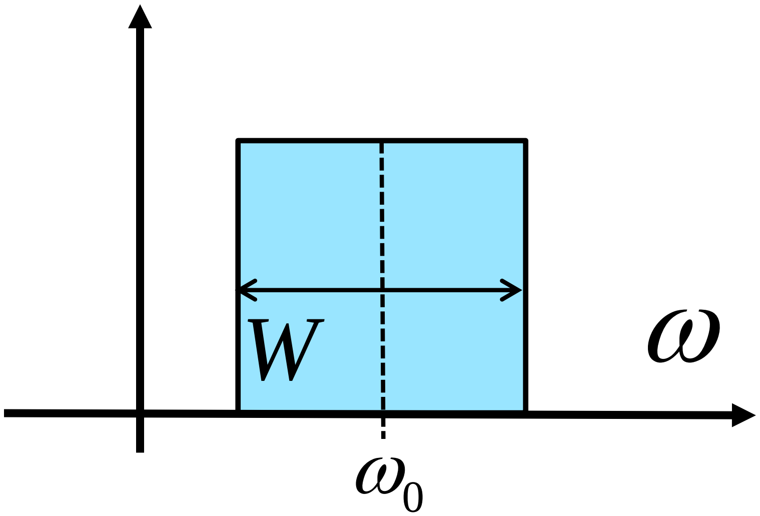
Transmitter



ILPF – Ideal Lowpass Filter (bandlimiting of information signal)
IBPF – Ideal Bandpass Filter (removing out-of-band frequencies)
MOD – Modulator (shifting information signal into channel bandwidth)
$x(t)$ – information signal, bandwidth ω_m
$\varphi(t)$ – modulated signal, bandwidth $W > \omega_m$
$c(t)$ – harmonic carrier, frequency $\omega_0 >> \omega_m$

Narrowband signals

$$\frac{W}{\omega_0} = \frac{B}{f_0} << 1(\approx 0)$$



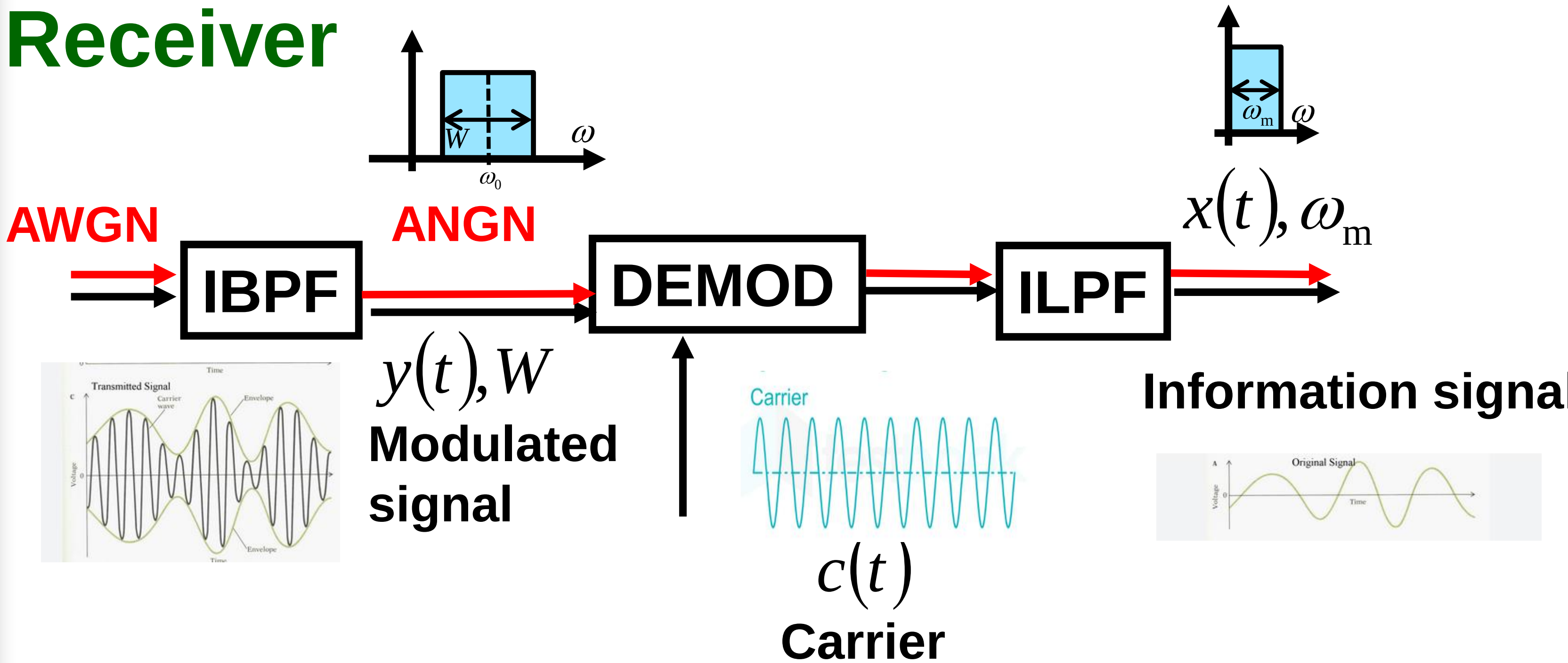
Stereo sound FM: B = 200 kHz, $f_0 \approx$ 100 MHz, B/f_0 = 0,002

CATV: B = 8 MHz, f0 ≈ 500 MHz, B/f0 = 0,016

SAT TV: B = 40 MHz, f0 ≈ 4 GHz, B/f0 = 0,01

Fiber transmission: III window 1550 nm ~ 200 THz!, window bandwidth 30 nm, B/f0 = 0,02

Receiver

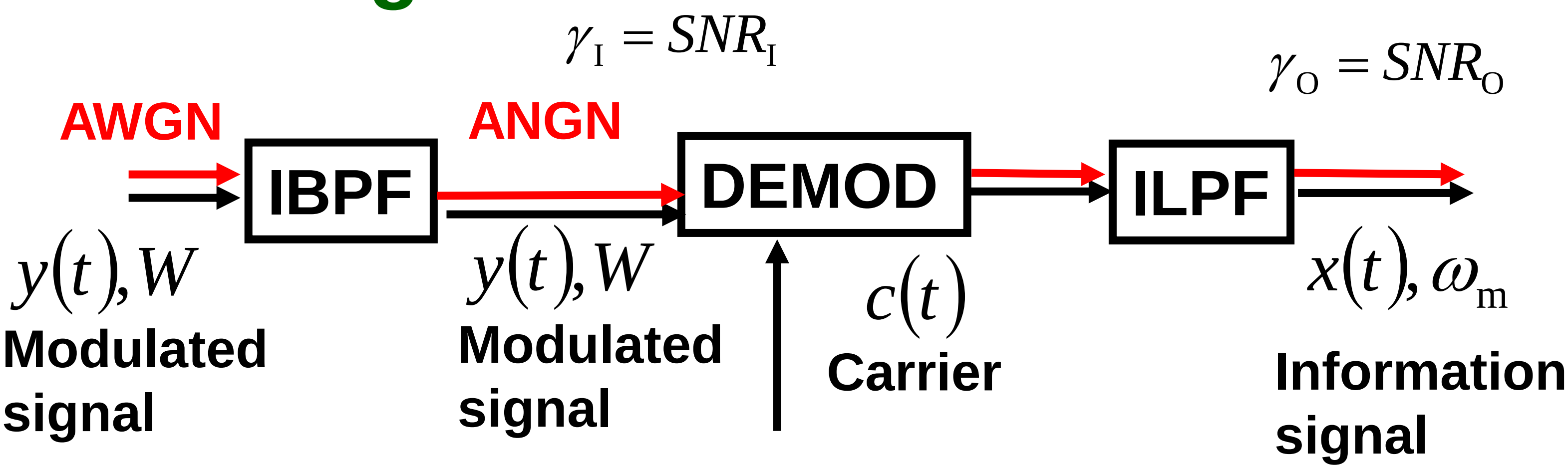


LPF – (Ideal) Lowpass Filter (removing out-of-band frequencies)
BPF – (Ideal) Bandpass Filter (removing out-of-band noise)
DEMOD – Demodulator (stripping information signal from the received signal)

Telecommunication system is narrowband, so the IBPF is narrowband and a narrowband noise (ANGN) disturbs the demodulator.

Bandwidth of the ILPF is equal to an information signal bandwidth so the output information signal is disturbed by an ANGN.

SNR in transmission systems - modulation gain



Noise affects all receiver components. Receiver immunity against noise is measured with a Signal to Noise Ratio $\gamma = SNR$ (Signal to Noise Ratio):

$\gamma = SNR$ = signal powrer to noise power ratio (general)

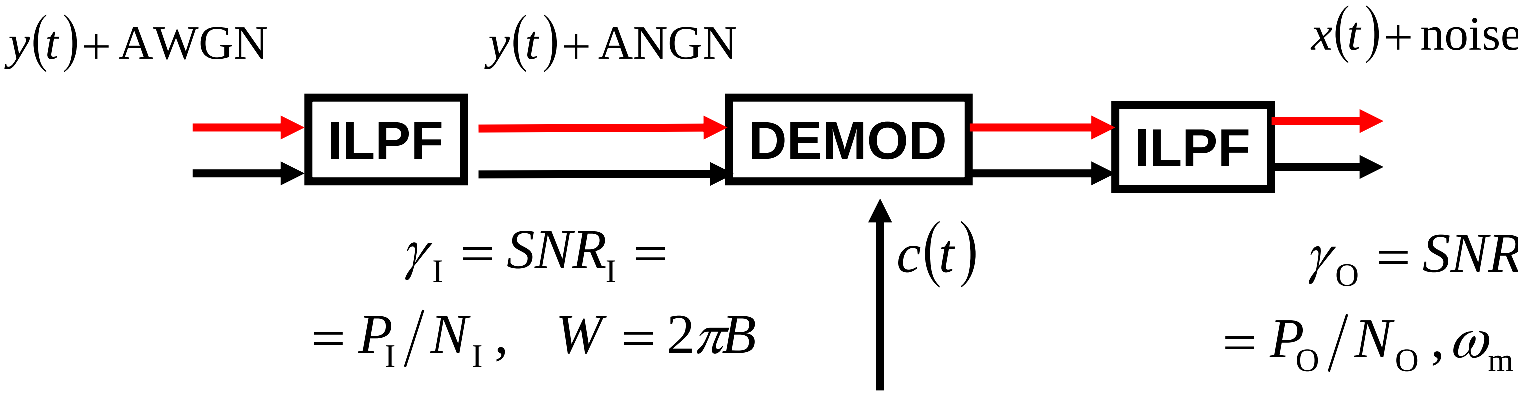
$\gamma_I = SNR_I$ = *SNR* on a demodulator input – demodulator noise environment

$\gamma_O = SNR_O$ = *SNR* on a system output – quality of an output signal

<u>Problems to be solved:</u>

- **model of ANGN (knowing spectrum is not sufficient),**
- **quest for a relation $\gamma_O = f(\gamma_I)$ – modulation gain, noise characteristic.**

Modulation gain, normalized bandwidth



$$\gamma_{\mathrm{I}} = SNR_{\mathrm{I}} = P_{\mathrm{I}}/N_{\mathrm{I}}, \quad W = 2\pi B$$

$$\gamma_{\mathrm{O}} = SNR_{\mathrm{O}} = P_{\mathrm{O}}/N_{\mathrm{O}}, \omega_{\mathrm{m}} = 2\pi f_{\mathrm{m}}$$

$$\text{normalized bandwidth } \kappa = W/\omega_{\mathrm{m}} = B/f_{\mathrm{m}}$$

$$\text{modulation gain} = g = \frac{SNR_{\mathrm{O}}}{SNR_{\mathrm{I}} \times \kappa} = \frac{\gamma_{\mathrm{O}}}{\gamma_{\mathrm{I}} \times \kappa}$$

Modulation gain g – outuput SNR_{O} given by a unit input SNR_{I} and a unit normalized bandwidth κ. Modulation gain is usually expressed in dB.

$$g[\mathrm{dB}] = \frac{\gamma_{\mathrm{O}}}{\gamma_{\mathrm{I}} \times \kappa} = \gamma_{\mathrm{O}}[\mathrm{dB}] - \gamma_{\mathrm{I}}[\mathrm{dB}] - 10\log\kappa[\mathrm{dek}]$$

Modulation gain – lowpass system

$y(t) + \mathrm{AWGN}$ $\qquad$ $y(t) + \mathrm{ANGN}$ $\qquad$ $x(t) + \mathrm{noise}$

$$\gamma_{\mathrm{I}} = SNR_{\mathrm{I}} = $$
$$= P_{\mathrm{I}} / N_{\mathrm{I}}, \quad W = 2\pi B$$

$$\gamma_{\mathrm{O}} = SNR_{\mathrm{O}} = $$
$$= P_{\mathrm{O}} / N_{\mathrm{O}}, \omega_{\mathrm{m}} = 2\pi f_{\mathrm{m}}$$

$$\kappa = \omega_{\mathrm{m}} / \omega_{\mathrm{m}} = f_{\mathrm{m}} / f_{\mathrm{m}} = 1$$

$$\gamma_{\mathrm{O}} = \gamma_{\mathrm{I}}$$
$$\kappa = 1$$

$$\text{modulation gain} = g = \frac{SNR_{\mathrm{O}}}{SNR_{\mathrm{I}} \times \kappa} = \frac{\gamma_{\mathrm{O}}}{\gamma_{\mathrm{I}} \times \kappa} = 1$$

Modulation gain is a measure how much a passband system is „better” or „worse” than a baseband system in terms of SNRs.

Noise Characteristics

$$g = \frac{SNR_{\mathrm{O}}}{SNR_{\mathrm{I}} \times \kappa} = \frac{\gamma_{\mathrm{O}}}{\gamma_{\mathrm{I}} \times \kappa}$$

$$\gamma_{\mathrm{O}} = g \times (\gamma_{\mathrm{I}} \times \kappa)$$

$$\gamma_{\mathrm{I}} \times \kappa = \frac{P_{\mathrm{I}}}{N_{\mathrm{I}}} \times \frac{W}{\omega_{\mathrm{m}}} = \frac{P_{\mathrm{I}}}{\eta W / \pi} \times \frac{W}{\omega_{\mathrm{m}}} = \boxed{\frac{P_{\mathrm{I}}}{\eta \omega_{\mathrm{m}} / \pi} = P_{\mathrm{I}} / N = \gamma}$$

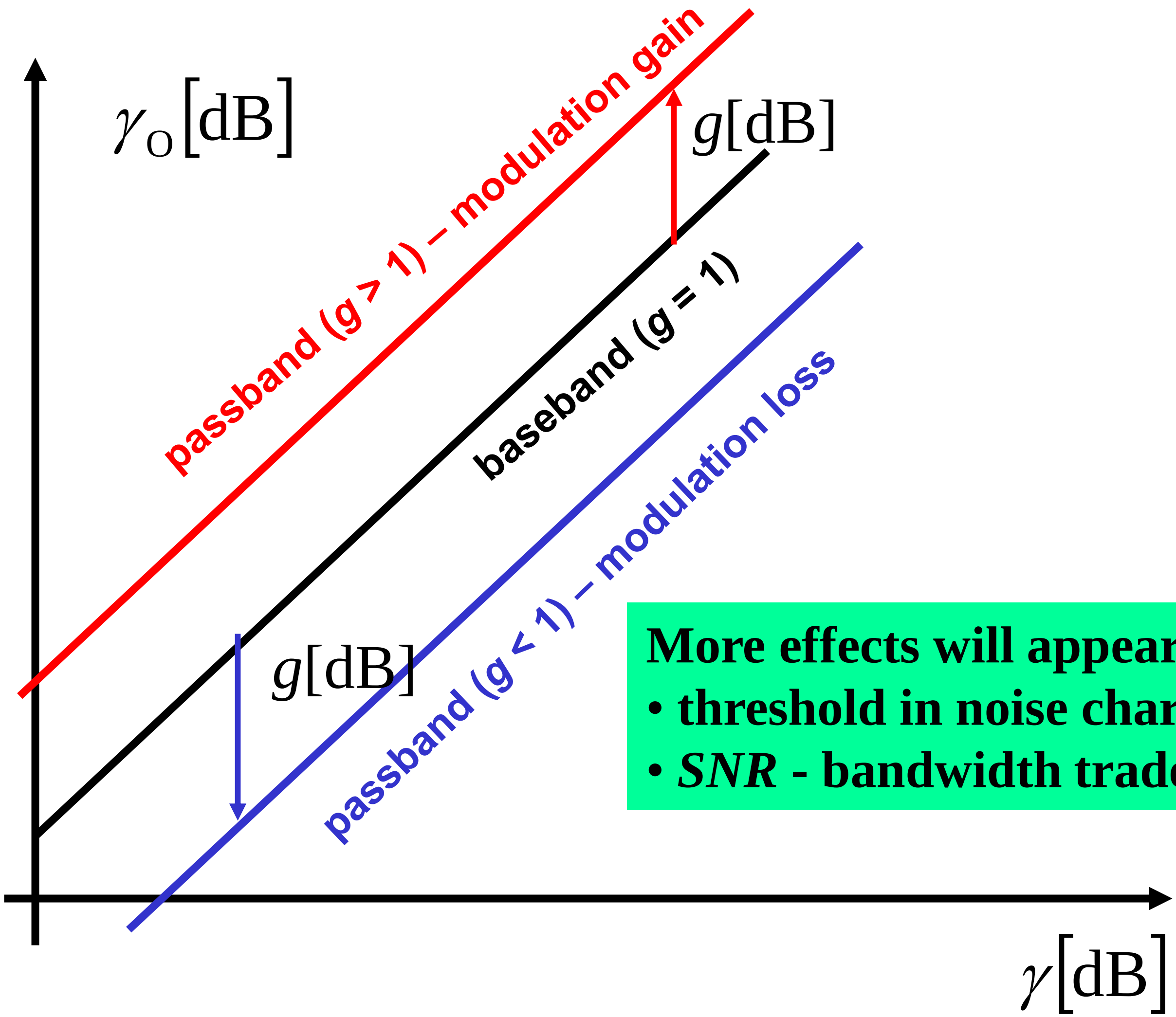
$$\gamma_{\mathrm{O}} = g \times \gamma$$

$$\gamma_{\mathrm{O}}[\mathrm{dB}] = g[\mathrm{dB}] + \gamma[\mathrm{dB}]$$

$$\boxed{\frac{P_{\mathrm{I}}}{\eta \omega_{\mathrm{m}} / \pi} = P_{\mathrm{I}} / N = \gamma}$$

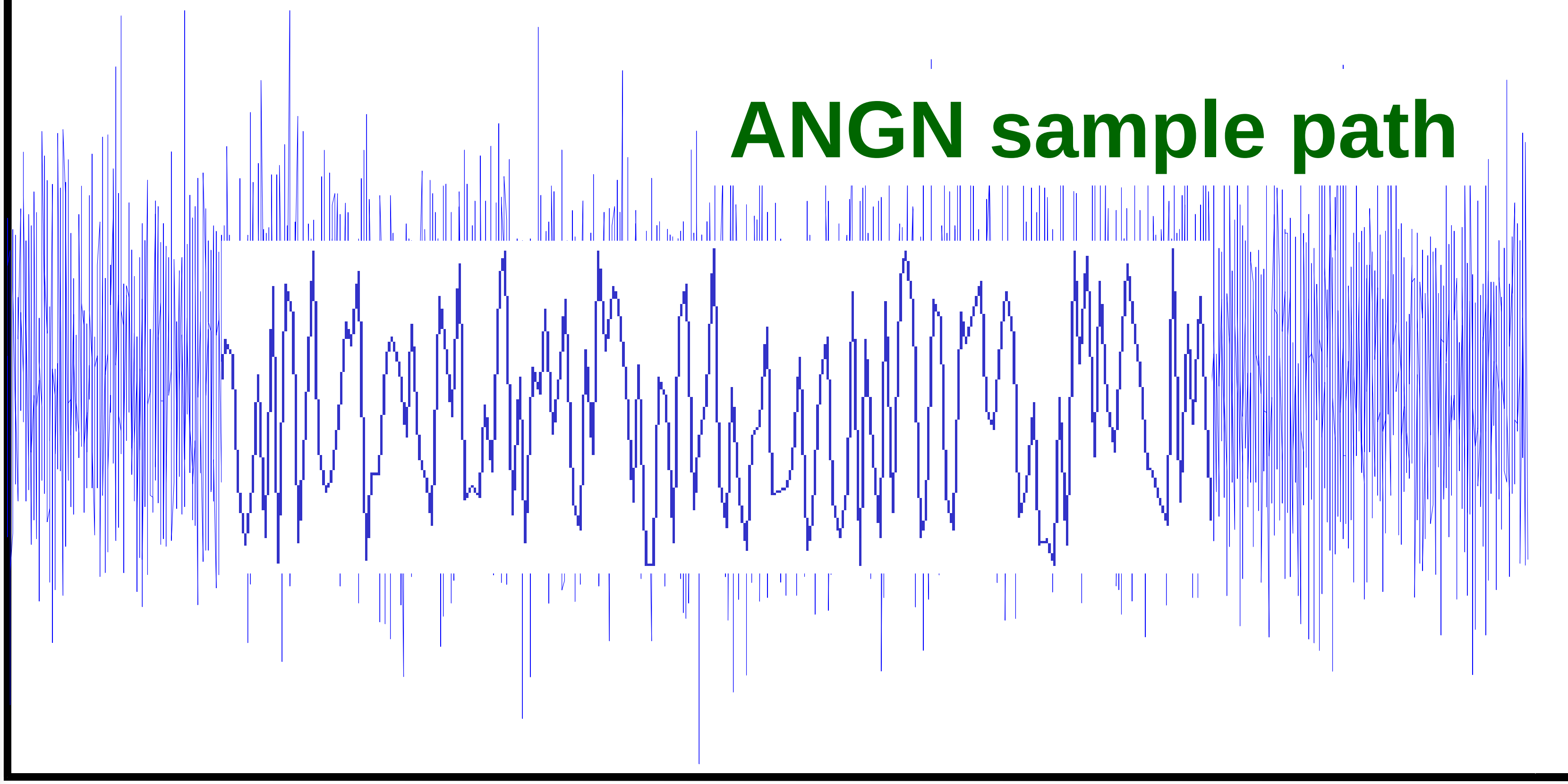
Signal to noise ratio γ of a modulated signal P_{I} to noise N present in a bandwidth ω_{m} of a modulating signal. Note that there is no point in the system with such an SNR.

Noise Characteristics



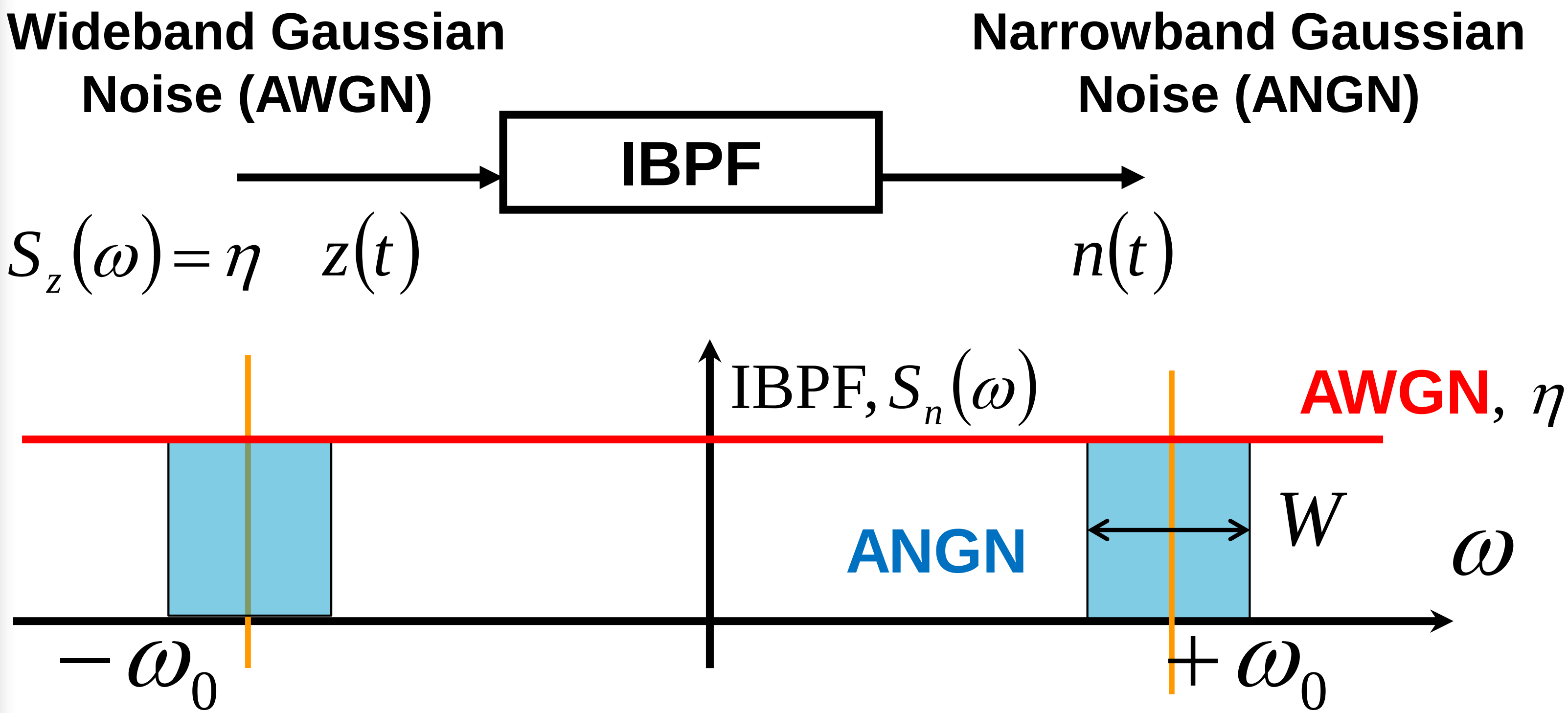
More effects will appear:
- **threshold in noise characteristcs**
- ***SNR* - bandwidth tradeoff**

Additive Narrowband Gaussian Noise (ANGN)



amplitude & frequency fluctuations

ANGN - Lowpass Representation



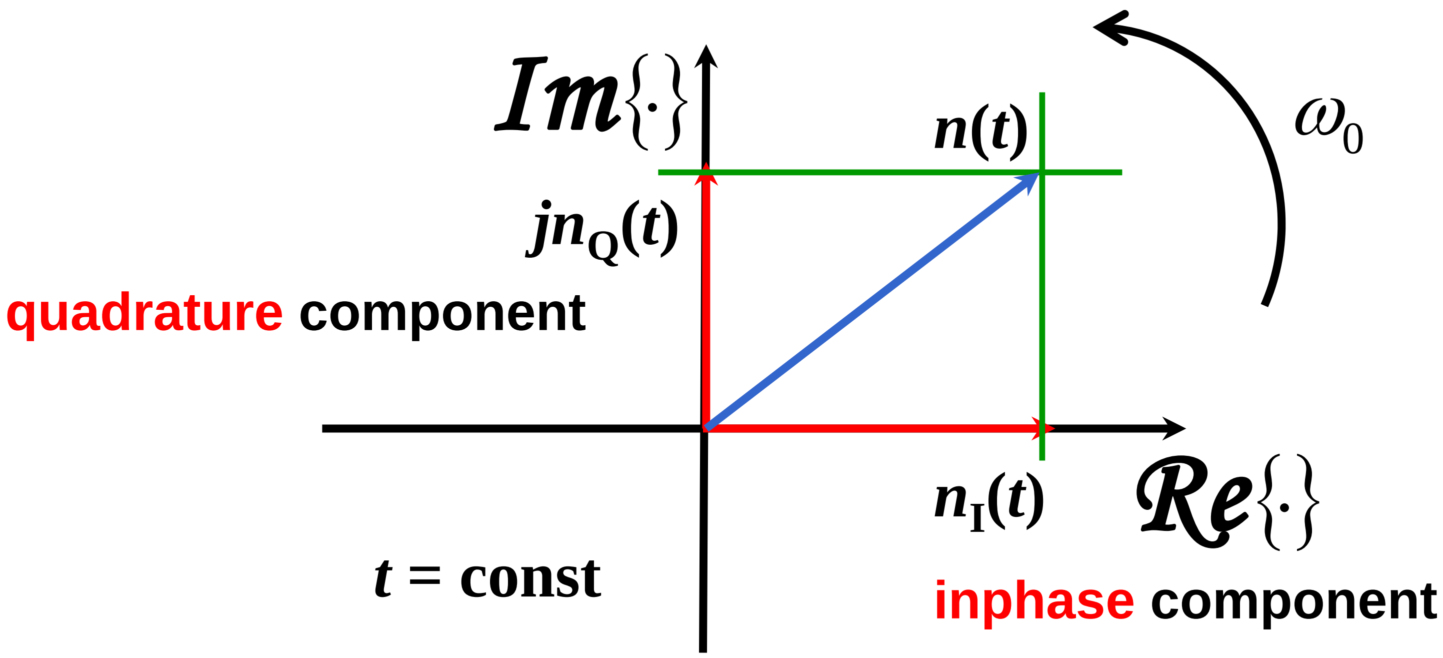
Lowpass representation of ANGN

$$n(t) = \underbrace{n_I(t)}_{\text{lowpass in-phase component}} \cos\omega_0 t - \underbrace{n_Q(t)}_{\text{lowpass quadrature component}} \sin\omega_0 t$$

ANGN – phasor diagram

$$n(t)=\mathcal{Re}\left\{\left[n_{\mathrm{I}}(t)+jn_{\mathrm{Q}}(t)\right]e^{j\omega_0 t}\right\}$$

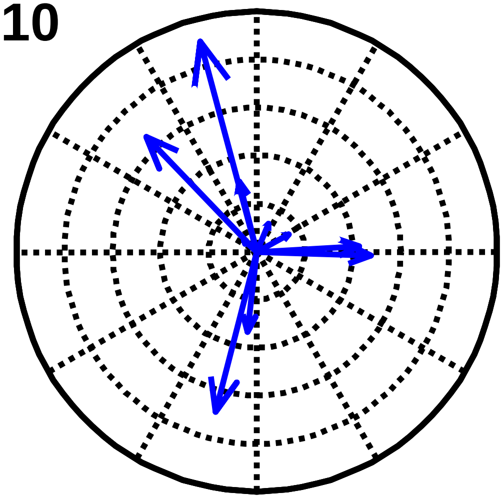
$$n(t)=n_{\mathrm{I}}(t)\cos\omega_0 t-n_{\mathrm{Q}}(t)\sin\omega_0 t$$

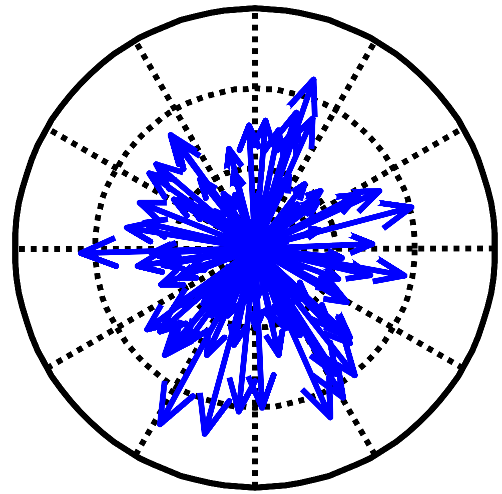


Both inphase and quadrature processes are modeled by a Gaussian i.i.d. r.v. which summation on the complex plane provides both amplitude and frequency (phase) random fluctuations.

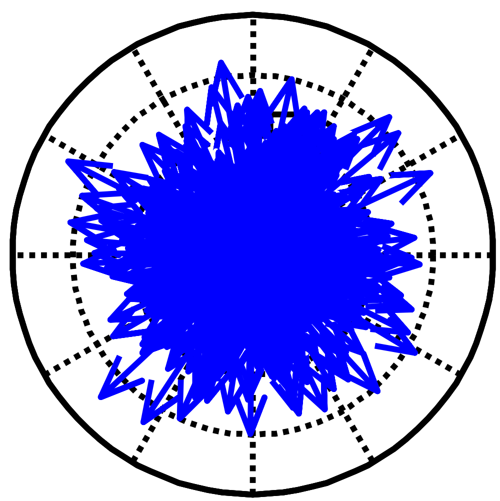
ANGN – phasor diagram (polar coordinates)

n = 10





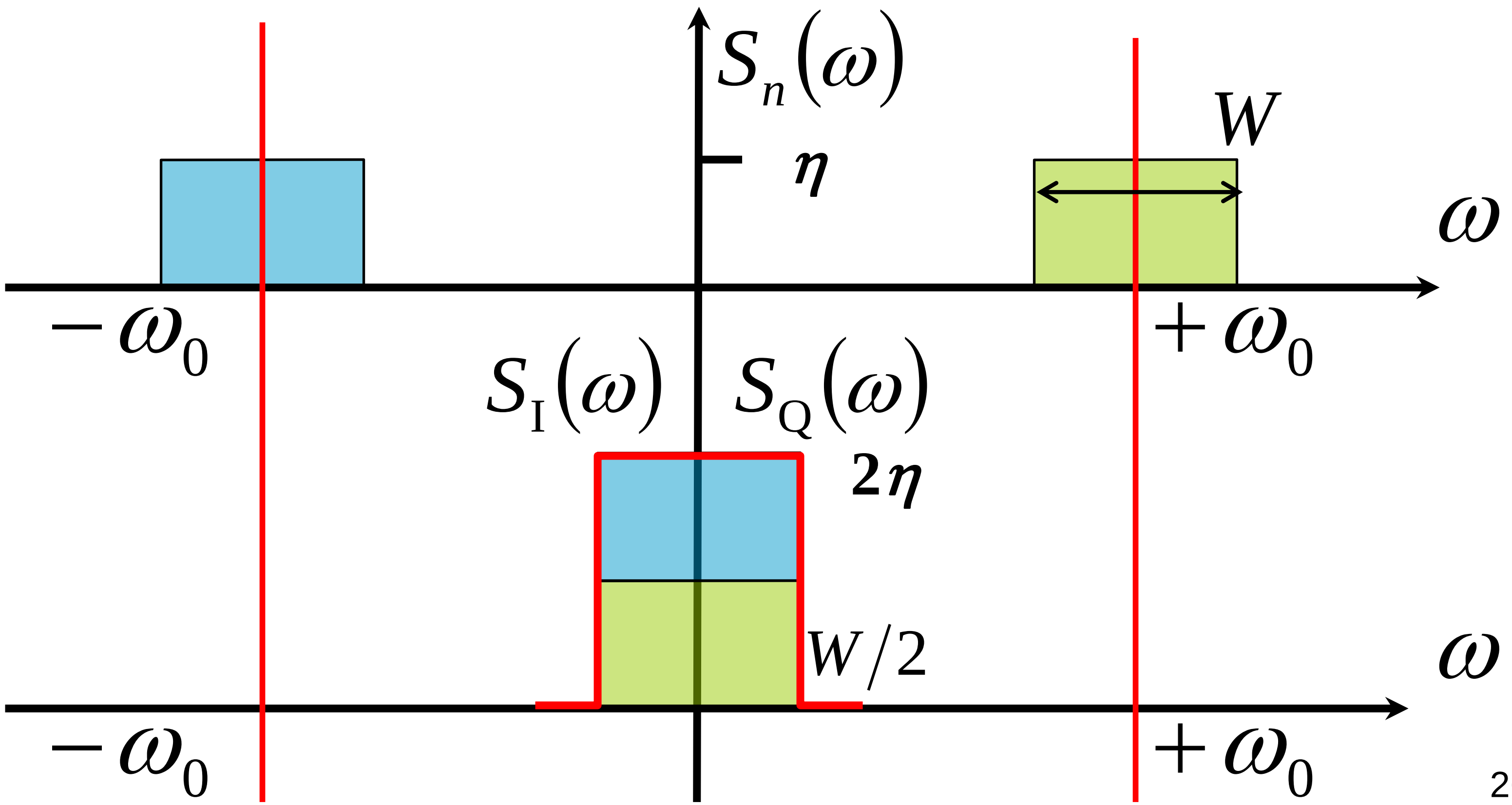
n = 100



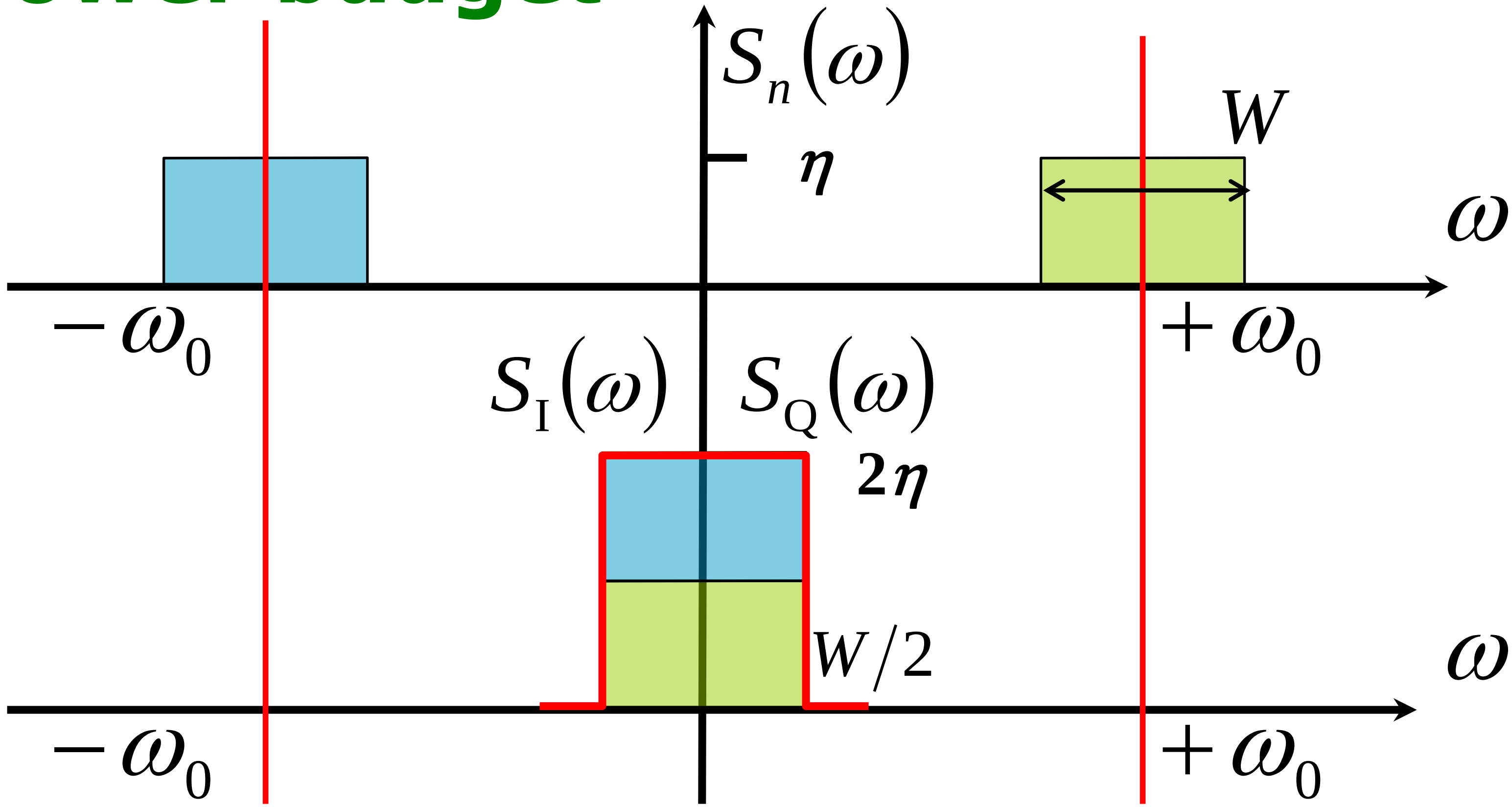
n = 1000

ANGN component spectra (double sideband case)

$$\left.\begin{matrix} S_{\mathrm{I}}(\omega) \\ S_{\mathrm{Q}}(\omega) \end{matrix}\right\} = \begin{cases} S_n(\omega+\omega_0)+S_n(\omega-\omega_0) \\ 0 \end{cases} = \begin{cases} 2\eta \\ 0 \end{cases}$$



ANGN – double sideband case – power budget



$$\overline{n^2} = \frac{1}{\pi}\int_W \eta d\omega = \frac{\eta W}{\pi}$$

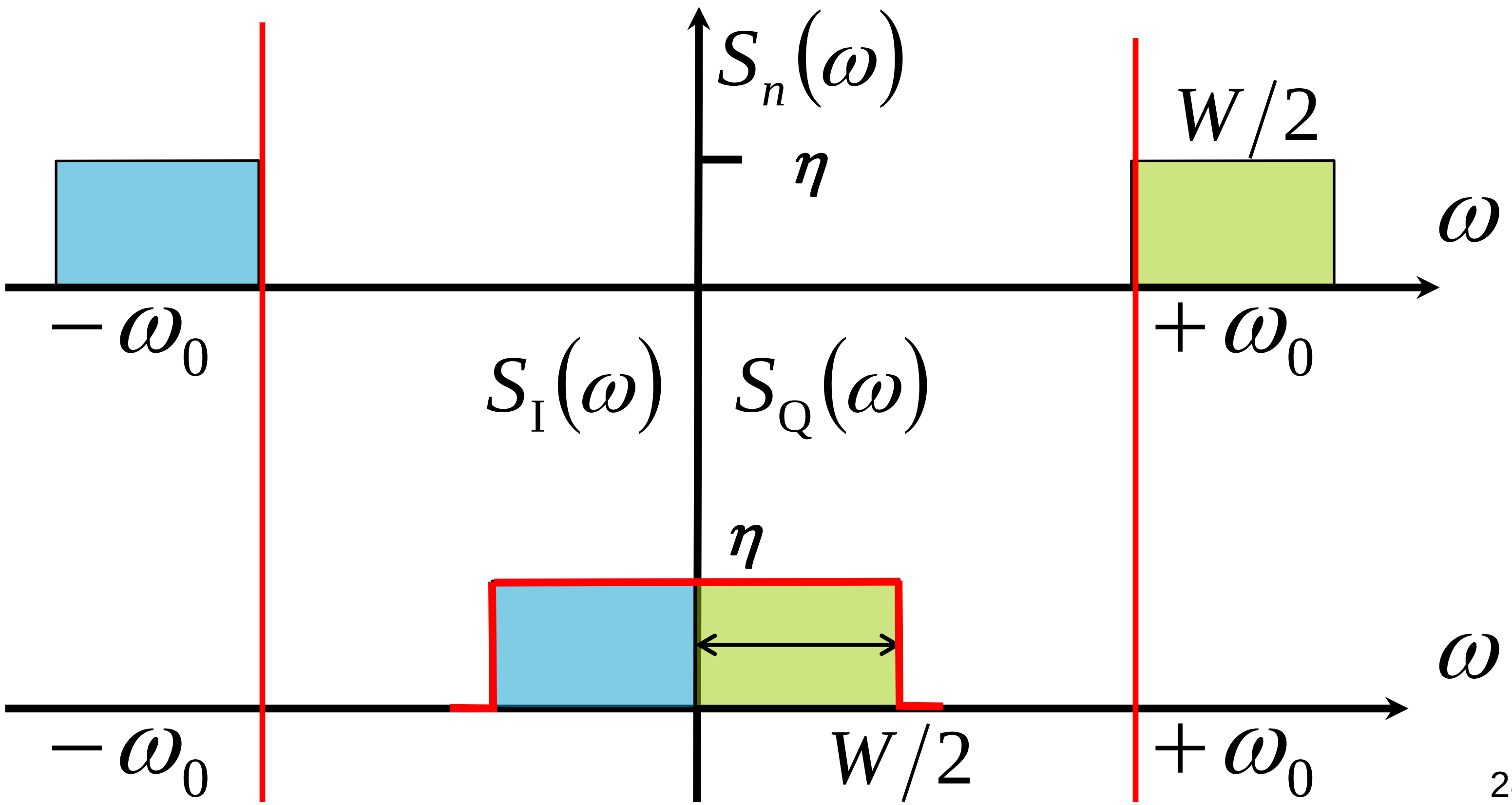
$$\overline{n_I^2} = \overline{n_Q^2} = \frac{1}{\pi}\int_0^{W/2} 2\eta d\omega = \frac{\eta W}{\pi}$$

$$n(t) = n_I(t)\cos\omega_0 t - n_Q(t)\sin\omega_0 t$$

$$\overline{n^2} = \frac{1}{2}\overline{n_I^2} + \frac{1}{2}\overline{n_Q^2}$$

ANGN component spectra (single sideband case)

$$\left.\begin{array}{l} S_{\mathrm{I}}(\omega) \\ S_{\mathrm{Q}}(\omega) \end{array}\right\} = \begin{cases} S_n(\omega+\omega_0)+S_n(\omega-\omega_0) \\ 0 \end{cases} = \begin{cases} 2\eta \\ 0 \end{cases} ; |\omega| \le \omega_0$$



Summary

Additive White Gaussian Noise:

- **simply adds to the information carrying signals**
- **has a flat spectrum across all frequencies**
- **fluctuates according to the normal p.d.f. (mean-squared variation = noise power)**
- **does not change its probabilistic nature when filtering (LPF, BPF)**
- **is modeled as a superposition of lowpass (inphase, quadrature) components which mimic both amplitude and frequency fluctuations**